Three-dimensional stabilization mechanism for the auroral Farley-Buneman instability

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Abstract–The auroral ionospheric E-region supports at least two types of plasma irregularities: the gradient drift instability and the Farley-Buneman instability. These are driven by the electron density gradient and the electric field, respectively. In this paper we study the effect of the three-dimensional auroral geometry on the saturation of the Farley-Buneman instability. For that purpose, an exact stationary solution to the two-fluid plasma model equations is derived. The stationary state thus obtained is used as an initial condition of a three-dimensional nonlinear fluid simulation. The algorithm is also described. In the auroral zone, the collision frequencies have gradients along the magnetic field. This implies that the phase velocity of the plasmons depends on the altitude, which in turns causes wave propagation at a nonzero aspect angle. These waves are more damped than exactly perpendicular ones. From this it is no surprise that our three-dimensional code can be used even when the electric field is near the Farley-Buneman threshold value. On the contrary, two-dimensional fluid algorithms blow up when the electric field is that large. We also conclude that the height profiles of the electric field, the electron density and the collision frequencies all affect coherent radar spectra, alongside with the local value of the electric field.

1. INTRODUCTION

The ionospheric E-region at about 100-115 km altitude is the layer where the auroral electrojet flows. The medium is characterized by well-magnetized electrons and neutral collision dominated ions. The neutral particle density is so high that the plasma is only weakly ionized although the electron density peaks at about this altitude. The horizontal electrojet currents are driven by horizontal electric fields caused by the ionosphere-magnetosphere coupling. Their strength depends on the degree of geomagnetic activity. The electric field drives the plasma more or less unstable, which causes propagating irregularities. Contemporary coherent radars are usually sensitive to a few meter wavelength perturbations.

These instabilities are stabilized by nonlinear processes, several of which have been studied by previous authors (for example *Haldoupis*, 1989, *Robinson* and *Honary*, 1990, *Haldoupis et al.*, 1990). A somewhat similar physical situation exists in the equatorial electrojet, too. That has been studied extensively both experimentally and theoretically (for example *Farley*,1985, *Ronchi et al.*, 1990). However, it is still a bit unclear how much the theoretical framework developed for the equatorial electrojet can be used in the auroral case.

Here we study three-dimensional effects that may stabilize the irregularities in the auroral electrojet. Some two-dimensional stabilization mechanisms are also discussed briefly. We have been led to three-dimensional mechanisms by the fact that the two-dimensional two-fluid model simulations simply explode when the electric field approaches the Farley-Buneman threshold value.

2. TWO-DIMENSIONAL PROCESSES

Consider a two-dimensional two-fluid model of the E-region plasma. Two-dimensionality means that there are no gradients along the magnetic field lines in the system. The linear dispersion relation for very small irregularities is approximately given by (see for example *Sudan et al.*, 1973)

$$\mathrm{Re}\,\omega = \frac{k_y E_0}{1+\psi}, \tag{1}$$

$$\mathrm{Im}\,\omega = \frac{1}{1+\psi}\{\psi_e[(\mathrm{Re}\,\omega)^2 - \mathbf{k}^2] + \frac{\psi_i k_y}{L\mathbf{k}^2}\mathrm{Re}\,\omega\}. \tag{2}$$

Here, the time and space dependence of the wave is assumed to be $exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$. We have chosen our coordinate system so that the z-axis points upwards, antiparallel to the geomagnetic field, and the horizontal electric field has only the x-component E_0. The x-component of the zeroth order electron density gradient is $1/L$. The collision and Larmor frequencies for electrons and ions are denoted by $\nu_{e,i}$ and $\Omega_{e,i}$, respectively, and we have defined $\psi = \psi_i\psi_e$ where $\psi_e = \nu_e/\Omega_e$ and $\psi_i = \nu_i/\Omega_i$. We measure spatial coordinates in units of the ion Larmor radius (roughly 2 m) and time in units of $1/\Omega_i$ (about 5 ms). The electric field is scaled so that the unit value corresponds to electron $\mathbf{E}\times\mathbf{B}$ drift velocity equal to the acoustic velocity C_s, which is given by $C_s^2 = k_B(T_e+T_i)/m_i$.

Note that unstable waves correspond to $\gamma = \mathrm{Im}\,\omega > 0$. If the flow angle ϑ (the angle between $\mathbf{k}$ vector and y-axis) is given, then waves with $k = |\mathbf{k}|$ less than k_{crit} are unstable, where the marginally unstable wavenumber k_{crit} is given by

$$k_{crit} = |\cos\vartheta| \sqrt{\frac{E_0 \psi_i (1+\psi)}{L\psi_e[(1+\psi)^2 - E_0^2 \cos^2\vartheta]}}. \qquad (3)$$

If L is large, then k_{crit} is small (only long wavelengths are unstable), unless the denominator of the square root expression becomes negative, in which case *all* wavelengths within a certain sector are unstable. The former case is called the gradient drift instability and the latter the Farley-Buneman instability. When the driving electric field is increased, a gradient drift case transforms continuously to a Farley-Buneman case (within the linear theory). Figs. 1-4 display the growth rate $\mathrm{Im}\,\omega$ as a function of k_x and k_y for four values of the driving electric field. Dimensionless units are used throughout Figs. 1-4. Negative growth rates (corresponding to damped waves) are displayed as zero. Below the threshold, the instability region in $\mathbf{k}$ plane is a finite "droplet". In the Farley-Buneman case the droplet opens up to a sector, which widens as the electric field strengthens. If $L = \infty$ the opening angle of the instability sector is given by $\vartheta_0 = \arccos \frac{1+\psi}{E_0}$, so the unstable Farley-Buneman sector exists only when $E_0 > 1 + \psi$. Already for $E_0 = 0.8(1+\psi)$ the unstable region is quite large, so that any two-dimensional fluid code has difficulties to converge. The limit $E_0 = 1 + \psi$ can be thought of as the point where the two-dimensional fluid model completely breaks down, in the mathematical sense.

For E_0 clearly less than $1 + \psi$ we are in the gradient drift regime (the droplet). Because the secondary electron density gradients (gradients associated with the irregularities, not the stationary profile) very easily exceed the zeroth order gradients $1/L$ and because gradients cause waves to propagate perpendicularly to them, a nearly isotropic turbulent quasi-steady state follows, which is identified with type II radar spectra (*Haldoupis*, 1989). This can be simulated rather easily by neglecting the ion inertia term as was done by *Keskinen et al.*(1979). Including the ion inertia leads to serious numerical difficulties especially with larger electric fields; that was attempted by *McDonald et al.*(1974). A theory of the turbulent k-spectrum in the gradient drift case was presented by *Sudan* (1977).

The main theoretical puzzle of the Farley-Buneman instability comes from the observational evidence that the phase speed $v_{phase} = \mathrm{Re}\,\omega/k_y$ is close to unity (or C_s in dimensional units) even if the driving electric field component exceeds the threshold $1 + \psi$ (*Farley*, 1985). A theory compatible with this observational fact was presented by *Sudan* (1983a, 1983b). He came up with the conclusion that the electrons collide with plasmons in such a way that the electron collision frequency and thus ψ is effectively increased. This adjusts the linear dispersion relation [Eq. (1)] so that the phase velocity does not exceed the threshold. It can be estimated that the so-called anomalous collisions are in fact more "frequent" than the classical electron-neutral collisions (*Sudan*, 1983b). Electron and ion heating effects can be incorporated into Sudan's theory in a self-consistent way (*Robinson*, 1986, *St.-Maurice*, 1987). A kinetic nonlinear model which produces Sudan's theory at the fluid limit was given by *Robinson* and *Honary*, (1990). It predicts that waves with small aspect angle may exist, but they are more easily stabilized by nonlinear mechanisms than zero aspect angle waves. On the other hand, *Schlegel* (1983) proved that in the linear theory the fastest growing modes have small but nonzero aspect angle.

Sudan's theory does not predict in detail the intensity or angular dependence of the wave spectrum (*Farley*, 1985). Nor does the theory make any difference (other than different collision frequencies etc.) between the auroral and the equatorial case. The latter is a general feature of two-dimensional instability models.

3. ALTITUDE COUPLING

In the auroral zone the ambient electron density and collision frequencies depend strongly on altitude, i.e. on the z-coordinate along the magnetic field. Thus also ψ and the phase speed [Eq. (1)] depend on altitude, assuming a constant horizontal electric field. On the other hand, the irregularities tend to be field-aligned which means that the system tolerates only weak gradients in the z-direction. Consider a wave structure that initially has the form

$$n(x,y,z) = N_0 exp(z)\, sin(k_x x)\, cos(k_y y). \qquad (4)$$

Because the wave phase speed depends on the altitude, the "wave tower" starts tilting, with the bottomside (for example roughly at 80 km altitude) propagating hardly at all. This causes growing gradients in the z-direction, which the system cannot tolerate indefinitely. We describe a three-dimensional simulation code to study the time development in later sections.

Natural vertical wavelengths in the system have a magnitude of $1/\psi_e \sim 150$ times a typical horizontal wavelength. If the latter is assumed to be 30 m, then the typical vertical gradients become 4.5 km, which is of the same order as the zeroth order scale height (for which we use the value 6 km in our calculation) in the ambient electron density and collision frequencies. This point will be discussed in more detail later in this paper. This rough estimation shows that three-dimensional altitude-coupling effects may well have relevance for the auroral situation. Note that in the equatorial geometry this argument is not valid.

4. STATIONARY STATE

In order to make a time-dependent simulation we need to specify the stationary state from which the instabilities grow. For two-dimensional situation such a state is trivial to construct, but in three dimensions it turns out to be rather cumbersome. We give here the resulting expressions in full form although they are used only to enter them into the computer. It should be stressed that the expressions that follow are not meant to be a realistic stationary ionospheric model, but an exact stationary solution to the fluid model equations. In other words: physically, a cruder model would suffice, but technically, we *must* use a model which satisfies our stationary equations *exactly*.

Again we use a coordinate system where the z-axis is upwards along the magnetic field, the y-axis is westward, and the x-axis points northward. Denote the magnetic inclination angle (the angle between the vertical and the magnetic field) by α, and use the value $\alpha = 15^o$. Assume that nothing depends on y, and that the y-component of the electric field vanishes. Denote the tangent of the inclination angle by s. Assume that the electron density n and the collision frequencies ν_e, ν_i only depend on altitude, so they are functions of $\tilde{z} = z + sx$ only. The electric field has two components, E_x and E_z. We take the ansatz that E_x is also only a function of the altitude, i.e. $E_x(z + sx)$.

The equations to be satisfied are the electron and ion momentum and continuity equations:

$$\mathbf{E} + \mathbf{v}_e \times \mathbf{B} + \theta_e \frac{\nabla n}{n} + \psi_e \mathbf{v}_e = 0 \tag{5}$$

$$\mathbf{E} + \mathbf{v}_i \times \mathbf{B} - \theta_i \frac{\nabla n}{n} - \psi_i \mathbf{v}_i = 0 \tag{6}$$

$$\nabla \cdot (n\mathbf{v}_e) = \nabla \cdot (n\mathbf{v}_i) = 0. \tag{7}$$

Here we have introduced the dimensionless temperature parameters θ_e and θ_i, defined by

$$\theta_e = \frac{m_e}{m_i}\left(\frac{u_e}{C_s}\right)^2, \quad \theta_i = \left(\frac{u_i}{C_s}\right)^2 \tag{8}$$

where u_i and u_e are the average thermal velocities of ions and electrons, respectively. Throughout this paper we assume that $\theta_e = \theta_i = \frac{1}{2}$. $\mathbf{B} = (0, 0, -1)$ in our dimensionless units.

Equations (5) and (6) are readily solved for $\mathbf{v}_e, \mathbf{v}_i$ to give

$$\mathbf{v}_e = \frac{1}{1+\psi_e^2}[\mathbf{E}_e{}' \times \mathbf{B} - \frac{1}{\psi_e}\hat{\mathbf{e}}_\mathbf{z} E_{ez}{}' - \psi_e \mathbf{E}_e{}'] \tag{9}$$

$$\mathbf{v}_i = \frac{1}{1+\psi_i^2}[\mathbf{E}_i{}' \times \mathbf{B} + \frac{1}{\psi_i}\hat{\mathbf{e}}_\mathbf{z} E_{iz}{}' + \psi_i \mathbf{E}_i{}'] \tag{10}$$

where

$$\mathbf{E}_e{}' = \mathbf{E} + \theta_e \nabla \ln n \tag{11}$$

$$\mathbf{E}_i{}' = \mathbf{E} - \theta_i \nabla \ln n \tag{12}$$

Now we are left with the determination of the electric field. For this we can use the continuity equations (7). However, because the electric field must be curl-free, it is not in general possible to satisfy both electron and ion continuity equations simultaneously. The reason to this "inconsistency" is that we have *prescribed* the electron density, not derived it self-consistently. However, we can satisfy the continuity equation of the current density

$$\nabla \cdot (n(\mathbf{v}_e - \mathbf{v}_i)) = 0 \tag{13}$$

and later check how rapidly $\nabla \cdot (n\mathbf{v}_e)$ changes the electron density. If this time scale is slow enough it can be compensated for by recombination processes. Another possibility would be to allow $\mathbf{E}$ to have curl, and later to demonstrate that the inductive part is small.

Finding $\mathbf{E}$ with these assumptions is a straightforward but lengthy calculation. The result can be expressed in terms of elementary functions, but it is not particularly simple in form. However, it can be easily manipulated using modern computer algebra systems. The solution requires exponential profiles for the collision frequencies and electron density, but the scale factors may be different. The solution contains three arbitrary constants c_1, c_2, c_3:

$$E_x(\tilde{z}) = \frac{1}{1+s^2\tilde{\beta}(\tilde{z})}[sc_2\Delta(\tilde{z}) - s\tilde{\gamma}(\tilde{z}) + c_3] \tag{14}$$

$$E_z(x, z) = \Delta(\tilde{z})f(x) - sE_x\tilde{\beta}(\tilde{z}) - \tilde{\gamma}(\tilde{z}) \tag{15}$$

where

$$f(x) = -\frac{c_1}{sb}e^{-sbx} + c_2 \tag{16}$$

$$b = -1/L - 1/\tilde{L} \tag{17}$$

$$n(\tilde{z}) = e^{\tilde{z}/L}, \quad \psi_i(\tilde{z}) = \psi_0 e^{-\tilde{z}/\tilde{L}}, \quad \psi_e(\tilde{z}) = e^{-\tilde{z}/\tilde{L}} \tag{18}$$

$$\Delta(\tilde{z}) = \frac{1}{n(\tilde{z})\alpha(\tilde{z})} \tag{19}$$

$$\alpha(\tilde{z}) = \frac{1}{\psi_i(\tilde{z})} + \frac{1}{\psi_e(\tilde{z})} \tag{20}$$

$$\beta(\tilde{z}) = \frac{\psi_i(\tilde{z})}{1+\psi_i(\tilde{z})^2} + \frac{\psi_e(\tilde{z})}{1+\psi_e(\tilde{z})^2} \tag{21}$$

$$\gamma(\tilde{z}) = \frac{s^2\theta_e\psi_e(\tilde{z})}{1+\psi_e(\tilde{z})^2} - \frac{s^2\theta_i\psi_i(\tilde{z})}{1+\psi_i(\tilde{z})^2} + \frac{\theta_e}{\psi_e(\tilde{z})} - \frac{\theta_i}{\psi_i(\tilde{z})} \tag{22}$$

$$\tilde{\beta}(\tilde{z}) = \beta(\tilde{z})/\alpha(\tilde{z}), \quad \tilde{\gamma}(\tilde{z}) = \frac{\gamma(\tilde{z})}{\alpha(\tilde{z})}\frac{n'(\tilde{z})}{n(\tilde{z})} \tag{23}$$

We have chosen the zero point of the z-coordinate so that $\psi_e(0) = 1$, which corresponds roughly to 70 km altitude. The symbol ψ_0 is the ratio ψ_i/ψ_e, which is assumed to be constant ($\psi_0 \approx 4000$). By choosing some other reference height h we can fix the constants $c_i, i = 1,2,3$ by demanding that $E_x(0) = E_x^{bot}, E_x(h) = E_x^{top}, E_z(0) = E_z^{bot}$. Here E_x^{bot}, E_x^{top} and E_z^{bot} are assumed to be given constants. This yields the following expressions for the constants c_i:

$$c_2 = \frac{1}{s[\Delta(0)-\Delta(h)]}\Big\{E_x^{bot} - E_x^{top} + s[\tilde{\gamma}(0)-\tilde{\gamma}(h)] + s^2[E_x^{bot}\tilde{\beta}(0) - E_x^{top}\tilde{\beta}(h)]\Big\} \tag{24}$$

$$c_3 = \frac{1}{\Delta(0)-\Delta(h)}\Big\{[1+s^2\tilde{\beta}(h)]E_x^{top}\Delta(0) - [1+s^2\tilde{\beta}(0)]E_x^{bot}\Delta(h) + s[\Delta(0)\tilde{\gamma}(h) - \Delta(h)\tilde{\gamma}(0)]\Big\} \tag{25}$$

$$c_1 = \frac{-1}{[1+s^2\tilde{\beta}(0)]\Delta(0)}\Big\{bs\big[E_z^{bot} + c_3 s\tilde{\beta}(0) + E_z^{bot}s^2\tilde{\beta}(0) - c_2\Delta(0) + \tilde{\gamma}(0)\big]\Big\} \tag{26}$$

The stationary solution is now complete. Now we can check how rapidly the electron density changes by calculating $\nabla\cdot(n\mathbf{v}_e)$ with the help of Eqs. (9), (14) and (15). The time scale turns out to be at least of the order of one hour for typical numerical values, which is very long compared with the instability phenomena under study.

5. THREE-DIMENSIONAL CODE

Having now an exact stationary solution to the two-fluid model equations, [given by Eqs. (14) and (15)] let us turn to the time-dependent case in order to do simulations. The basic equations are (we neglect electron inertia)

$$\frac{\partial n}{\partial t} = -\nabla\cdot(n\mathbf{v}) \tag{27}$$

$$\nabla\cdot(n\mathbf{v}_e) = \nabla\cdot(n\mathbf{v}) \tag{28}$$

$$\mathbf{E} + \mathbf{v}_e\times\mathbf{B} + \theta_e\frac{\nabla n}{n} + \psi_e\mathbf{v}_e = 0 \tag{29}$$

$$\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = \mathbf{E} + \mathbf{v}\times\mathbf{B} - \theta_i\frac{\nabla n}{n} - \psi_i\mathbf{v} \tag{30}$$

where $\mathbf{v} = \mathbf{v}_i$. The meaning of other symbols has been explained in the previous chapter. We expand the total electric field $\mathbf{E}$ as $\mathbf{E} = \mathbf{e}_0 - \nabla\phi$ and define the effective potential $u = \phi - \theta_e \ln n$. Here $\mathbf{e}_0$ is the stationary solution electric field, denoted by $\mathbf{E}$ in the previous chapter. We choose to express our equations in terms of u rather than in terms of ϕ only because it is a bit more convenient that way. We eliminate $\mathbf{v}_e$ from Eq. (29) and substitute it to Eq. (28.) In this way, we obtain an elliptic equation for the effective potential u. Let us write down our set of equations at this stage:

$$\frac{\partial n}{\partial t} = -\nabla\cdot(n\mathbf{v}) \tag{31}$$

$$\frac{\partial\mathbf{v}}{\partial t} = \mathbf{v}\times\mathbf{B} - \psi_i\mathbf{v} + \mathbf{e}_0 - \nabla u - \theta\nabla\ln n - \mathbf{v}\cdot\nabla\mathbf{v} \tag{32}$$

$$n\psi_e\psi_q\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{n}{\psi_e}\frac{\partial^2 u}{\partial z^2} + \left[\frac{\partial}{\partial x}(n\psi_e\psi_q) - \psi_q\frac{\partial n}{\partial y}\right]\frac{\partial u}{\partial x} + \left[\psi_q\psi_e\frac{\partial n}{\partial y} + \frac{\partial}{\partial x}(n\psi_q)\right]\frac{\partial u}{\partial y} + \frac{\partial}{\partial z}\left(\frac{n}{\psi_e}\right)\frac{\partial u}{\partial z} = \nabla\cdot(n\mathbf{v}) - \nabla\cdot(n\mathbf{v}_{e0}) \tag{33}$$

where $\theta = \theta_i + \theta_e$, $\psi_q = 1/(1+\psi_e^2)$ and $\mathbf{v}_{e0}$ is given by Eq. (9) with $\mathbf{E}_e{}'$ set equal to $\mathbf{e}_0$.

Looking only at the second order u-derivative terms in Eq. (33), the natural horizontal/vertical scaling is simply $\Delta X \simeq \psi_e\Delta Z$, where ΔX and

ΔZ temporarily denote typical horizontal and vertical wavelengths, respectively. Compare with *Haldoupis* (1989), Eq. 4 for the influence of the aspect angle in the linear dispersion relation. If we assume that a typical aspect angle δ is just given by $\tan\delta \approx \delta \simeq \Delta X/\Delta Z$, we obtain $\delta \simeq 0.5^o$ for $\psi_e \approx 0.007$. This rough estimate for the aspect angle is not in disagreement with our simulation results (presented in the next section). Recall that ψ_e itself decreases exponentially with altitude, so $\Delta X/\Delta Z$ is likely to more close to unity at lower altitudes. Other terms in Eq. (33), particularly the last one on the left-hand-side, introduce $\mathbf{k}$-dependence on $\Delta X/\Delta Z$. These complications make it difficult to construct more accurate analytical estimations of the horizontal/vertical scaling; this is one of the reasons why the numerical simulation technique is useful here.

Because the relevant parameters in Eq. (33) have exponential height dependence, one could consider using a non-uniform mesh in the z-direction, something like $\Delta z \sim exp(z/L)$. This approach has a drawback that only a minority of mesh points lay in the upper part of the simulation box, where the interesting things are taking place. We use a uniform mesh in this work.

The continuity equation [Eq. (31)] is discretized at nth timestep as

$$\begin{aligned} \frac{n^{(n+\frac{1}{2})} - n^{(n-\frac{1}{2})}}{\Delta t} = \\ -\frac{1}{2}\left[n^{(n+\frac{1}{2})} + n^{(n-\frac{1}{2})}\right] \nabla\cdot\mathbf{v}^{(n)} \\ -\mathbf{v}^{(n)}\cdot\nabla n^{(n-\frac{1}{2})} \end{aligned} \tag{34}$$

which is easily solvable for $n^{(n+\frac{1}{2})}$. Here the upper indices refer to timesteps. The momentum equation [Eq. (32)] is handled similarly,

$$\begin{aligned} \frac{\mathbf{v}^{(n+1)} - \mathbf{v}^{(n)}}{\Delta t} = \frac{1}{2}\left[\mathbf{v}^{(n+1)} + \mathbf{v}^{(n)}\right] \times\mathbf{B} \\ -\frac{1}{2}\psi_i\left[\mathbf{v}^{(n+1)} + \mathbf{v}^{(n)}\right] + \mathbf{F} \end{aligned} \tag{35}$$

where $\mathbf{F} \equiv \mathbf{e}_0 - \nabla u - \theta\nabla\ln n - \mathbf{v}\cdot\nabla\mathbf{v}$. This time integration scheme is not completely symmetric but it is nevertheless dominated by the symmetric parts. Making it more symmetric would require solving a sparse linear system at each timestep, a path we decided not to take. The elliptic equation [Eq. (33)] is solved after each timestep by the successive over-relaxation method (SOR; see *Press et al.*, 1988) by taking u from the previous timestep as an initial guess. The stationary state solution given by equations (14) and (15) plus a small perturbation of the form (4) is used as the initial value.

We have found that the magnetic inclination angle α has practically no effect, so we could equally well think that $\mathbf{B}$ is vertical. This is because our simulation box is very tall (over 10 km) and narrow (20 m x 20 m). This choice of scaling makes the coefficients of the elliptic equation as smooth as possible, i.e. makes the best numerical sense. Our boundary conditions are periodic in x and y, Dirichlet at the bottom and Neumann at the top. The boundary conditions are imposed after the stationary solution has been subtracted. This means that for example at the top the vertical derivative of the electron density equals the vertical derivative of the stationary solution. The number of grid points used in the example run was 32 in x, 32 in y, and 44 in the z-direction.

Our example simulation run contained 3000 timesteps and lasted about 0.2 physical seconds. It took about one hour of Cray X-MP CPU time. The performance of the code was about 40000 grid points per CPU second per timestep. This relatively low figure is explained by the fact that the average number of SOR iterations per timestep (in the elliptic solver) was about 10. See Table 1 for more information.

A more unpleasant thing is that the run terminated because the elliptic solver did not converge. This is a general feature of many fluid codes, especially those that deal with quasineutral plasmas. This is mainly because the elliptic equation to be solved [Eq. (33)] is far more complicated than for example the simple Poisson equation used in ordinary particle simulations of plasma physics. Because the elliptic equation contains first-order derivatives, it is not always diagonally dominated enough to behave well in an iteration method like SOR. In our program we actually used a slight modification of SOR where the best convergence parameter ω is experimentally searched during the run.

One could also consider doing a particle simulation of this situation (*Machida* and *Goertz*, 1988). The difficulties in such an approach are different from the fluid code: the code can be made numerically unconditionally stable, but compromises must be made: the system becomes noisy, the Debye length easily becomes unrealistically large or the simulation box very small. Furthermore, feasible three-dimensional particle codes are still almost impossible to run in existing computers. Quasineutral particle codes could solve the Debye length problem but the elliptic equation to be solved would no longer be Poisson type.

6. RESULTS

In Fig. (5) we display a time series of the three-dimensional simulation run. The electron density variation (the total density minus the initial value) is plotted as a color chart on three perpendicular planes. The uppermost horizontal plane is located at the electrojet altitude and it is the plane where the two-dimensional simulations have been performed.

The six frames in Fig. (5) are taken at equal time intervals, when the physical time runs from 0 to about 0.2 sec. The actual number of frames stored and studied was 30 instead of six. The system has been initialized with the stationary solution plus a small perturbation as given by Equation (4). The color scaling is the same for all six frames.

In the first frame the initial perturbation has grown in amplitude by about a factor of ten and is now of the order of one percent on the topside. The structure has propagated a bit along the y-axis but is not notably distorted. Arriving to the second frame, quite a lot has happened (this can be seen on intermediate frames not shown in this paper). The tilting began suddenly after the first frame, and the original sinusoidal xy-dependence has been significantly distorted. The aspect angle as estimated from the figure is now about 0.05^o. It *appears* to be roughly 30^o because the height/width ratio of the simulation box is over 600. Moving on to frame 3, the tilting continues (aspect angle grows), and the system somehow recovers the original sinusoidal horizontal waveform. At one time between frames 2 and 3, the waveform was even more distorted than in frame 2. In frame 4, we see the first signs of shorter wavelengths (developing turbulence) in the horizontal plane. These signs can also be seen in frames 5 and 6. In some of the intermediate frames between frames 4 and 5 the shorter wavelengths seem to be stronger. Towards the end of the run the aspect angle grow continuously, approaching half a degree at the end. The run terminates to an error message from the elliptic solver, because the vertical wavelengths become too short compared with the grid spacing.

During our simulation, the aspect angle of the main wave was growing more or less continuously. Proper turbulence did not develop, but some signs of smaller-scale waves were seen in the upper part of the simulation box. The absence of field-perpendicular turbulence should be contrasted with earlier two-dimensional gradient drift instability simulations (*Keskinen et al.*, 1979), where strong perpendicular turbulence developed even for driving electric fields less than 30 percent of the threshold value. One explanation to this may be that the field-perpendicular electron density scale length was large in our simulation (as it usually is in the auroral case) because the density structure was assumed to be horizontal; it varied between 20 km and infinity between different runs as the magnetic inclination angle was varied and the total density scale was kept at 6 km. The results did not notably depend on the inclination angle. But in our view, the altitude coupling mechanism is at least partially responsible for this lack of turbulence. A final point is that, evidently, some sort of vertical mixing (or turbulence) should occur if the three-dimensional runs could be followed further in time and that this turbulence could partly turn into field-perpendicular smaller scale irregularities as well. But also in the last case, the observed perpendicular turbulence follows after the parallel one. Recall the scaling of our simulation box (over 10 km high, 20 meters wide) and the fact that the parallel turbulence can not really be observed with radars because the wavelengths are extremely long.

The present simulation code produces, in our opinion, reliable results whenever its elliptic solver converges. Because it does not always converge, we call it preliminary.

7. SUMMARY AND CONCLUSIONS

Three-dimensional two-fluid simulations of the E-region irregularities have been performed. They show three-dimensional altitude coupling in the auroral ionosphere that significantly damps the otherwise extremely unstable Farley-Buneman waves. For simulation purposes, we derived an analytic stationary solution to the two-fluid model, applicable to the auroral geometry with possibly nonzero magnetic inclination angle. The altitude coupling process can only work in the auroral geometry, where the parameters have relatively strong gradients along the magnetic field. Another view of the same mechanism is to note that the difference of phase speed in different layers causes the waves to have nonzero aspect angle (or causes them to "tilt"), which in turn damps them according to the linear theory (*Haldoupis*, 1989, *Robinson* and *Honary*, 1990). The sign of the nonzero aspect angle is such that the waves try to penetrate into a region of decreasing electron density and increasing collision frequencies (i.e. downwards).

We also conclude that the ionosphere responds as a whole to the "supersonic" electric field, although the Farley-Buneman threshold is possibly exceeded only in a certain layer. At sufficiently low altitudes, we can always find a region where the neutral collisions make ψ so large that the threshold is not exceeded.

In the future we want to find a more successful elliptic solver that would allow one to continue the run until the vertical gradients no longer grow but the vertical mixing phase (presumably) begins. It is likely that a larger number of vertical grid points would also help, because the code diverges only after the vertical wavelength approaches the vertical grid spacing. The heating of the electrons (possibly ions, too) should also be included self-consistently, along the lines shown by *Robinson* (1986).

Another question not addressed yet is what happens *above* the electrojet altitude (in the sense of coupling of different layers). In our simulation, we circumvented the problem simply by imposing Neumann boundary conditions on the topside.

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Table 1. Various 3D run parameters.

Δt	0.06 ms	Timestep
Δx	0.6 m	Grid spacing in x-direction
Δy	0.6 m	Grid spacing in y-direction
Δz	300 m	Grid spacing in z-direction
L_x	19.2 m	Simulation box length in x-direction
L_y	19.2 m	Simulation box width in y-direction
L_z	13.2 km	Simulation box height in z-direction
z_0	86 km	Simulation box bottom altitude
E_x^{bot}	0	Stationary E_x at 70 km altitude
E_x^{top}	1.2	Stationary E_x at 110 km altitude
E_z^{bot}	0	Stationary E_z at 70 km altitude
L	6 km	El. density (vertical) scale length
$\tilde{L}$	6 km	Collision frequency scale length
xperturb	0.005	Initial x-perturbation amplitude
yperturb	0.01	Initial y-perturbation amplitude
timesteps	2976	Number of timesteps taken
ψ_e range	0.0077 .. 0.073	
ψ_i range	30.8 .. 292	
ψ range	0.23 .. 21	
$(1+\psi)$ range	1.23 .. 22	So Farley-Buneman threshold is ≈ 1.23 at the top
E_x range	1.1936 .. 1.19993	
E_z range	-0.000233 .. -0.000170	

Fig. 1. Growth rate for driving field $e_0 = 0.8$
Arbitrary linear scale.

Fig. 2. Growth rate for driving field $e_0 = 0.9$

Fig. 3. Growth rate for driving field $e_0 = 1.0$
(just below the threshold)

Fig. 4. Growth rate for driving field $e_0 = 1.1$
(just above the threshold)